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Propagation of non-axisymmetric waves in an infinite soft electroactive hollow cylinder under uniform biasing fields

Y.P. Su[a], H.M. Wang[a], C.L. Zhang[a], W.Q. Chen[a,b,c,d,*]

[a] Department of Engineering Mechanics, Zhejiang University, Hangzhou 310027, China
[b] State Key Lab of CAD & CG, Zhejiang University, Hangzhou 310058, China
[c] Soft Matter Research Center (SMRC), Zhejiang University, Hangzhou 310027, China
[d] Key Laboratory of Soft Machines and Smart Devices of Zhejiang Province, Zhejiang University, Hangzhou 310027, China

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ABSTRACT

Based on Dorfmann and Ogden's nonlinear theory of electroelasticity and the associated linear incremental theory, the non-axisymmetric wave propagation in an infinite incompressible soft electroactive hollow cylinder under biasing fields is investigated. The biasing fields are uniform, including an axial pre-stretch and a radial stretch in the plane perpendicular to the axis of the cylinder as well as an axial electric displacement. Such biasing fields make the originally isotropic electroactive material behave during its incremental motion like a conventional transversely isotropic piezoelectric material, hence greatly facilitating the following analysis. The three-dimensional equations of wave motion in cylindrical coordinates are derived and exactly solved by introducing three displacement functions. The exact solution is expressed in terms of Bessel functions, and explicit frequency equations are presented in different cases. For a prototype nonlinear model of electroactive material, numerical results are given and discussed. It is found that the initial biasing fields as well as the geometrical parameters of the hollow cylinder have significant influences on the wave propagation characteristics.

1. Introduction

Soft electroactive materials are smart materials, which may be produced by embedding electroactive particles in a rubber-like matrix such as silica gel and silicone rubber (Bossis et al., 2001). They have attracted considerable interests and are widely used to develop high-performance mechanical devices such as actuators and artificial muscles because of their rapid response and large deformation under electrical stimulus (Anderson et al., 2012; Henann et al., 2013).

Nonlinear analysis of soft electroactive materials or structures is quite complex due to the strong nonlinearity as well as the electromechanical coupling. The formulation of the general nonlinear theory of electroelasticity dates back to the 1950s. Toupin (1956, 1963) first established the theories governing the static and dynamic responses of elastic dielectrics. Tiersten (1971) later extended Toupin's study to the case with thermal effect. The nonlinear interactions between the mechanical and electromagnetic fields are well expounded in the books by Landau and Lifshitz (1960), Nelson (1979), and Maugin (1988), to name a few. Theoretical development of the nonlinear theories of electroelasticity has been revived in the recent decade (Dorfmann and Ogden, 2005, 2006; McMeeking and Landis, 2005; Mockensturm and Goulbourne, 2006; Bustamante et al., 2009; Suo, 2010) since new soft electroactive materials have been produced, indicating a very tempting prospect of applications.

The study on waves in electroactive materials not only presents significant theoretical interests but also is of specific practical importance. Chai and Wu (1996) applied the Lothe–Barnett's integral formalism to the study of surface waves in a prestressed piezoelectric material. The initial stress effect on the reflection coefficients of waves in a prestressed piezoelectric half-space was discussed in a recent paper by Singh (2010). Based on the nonlinear framework for electroelasticity (Dorfmann and & Ogden, 2005, 2006) and the associated linear incremental theory (Dorfmann and & Ogden, 2010b), Dorfmann and Ogden (2010a) analyzed the plane waves propagating in a homogeneously deformed electroactive material and the surface waves in a homogeneously deformed half-space of incompressible electroactive material. Axisymmetric waves in pre-stretched incompressible soft electroactive cylinders were examined in an exact manner by Chen and Dai (2012), also based on the theoretical framework suggested by Dorfmann and Ogden. In a more recent paper, Su and Chen (2014) extended Chen and Dai's work to a cylindrical shell and further considered the influence of the electric field exterior to the shell. Almost simultaneously, Shmuel et al. (2012)

* Corresponding author at: Department of Engineering Mechanics, Zhejiang University, Hangzhou 310027, China. Tel.: +86 57187951866; fax: +8657187951866.
E-mail address: cwqcmx@gmail.com, chenwq@zju.edu.cn (W.Q. Chen).

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showed a strong effect of the biasing fields on the propagation of Rayleigh–Lamb waves in dielectric layers. Axisymmetric waves in dielectric elastomer tubes under biasing fields were also studied by Shmuel and deBotton (2013), where the biasing field is produced by applying a voltage difference between the inner and outer surfaces of the shell. This is quite different from that in Chen and Dai (2012) and Su and Chen (2014), which actually results in nonuniform biasing fields, and makes it impossible to obtain exact solutions.

In this paper, we aim at developing an exact analysis of non-axisymmetric waves in an infinite soft electroactive hollow cylinder subjected to uniform pre-stretch and/or biasing electric field. This is an extension of our previous works mentioned above, where only the simple axisymmetric case was considered. For the purpose of analysis, the theories of nonlinear electroelasticity and linear incremental field proposed by Dorfmann and Ogden (2005, 2006, 2010b) are briefly reviewed. As in Chen and Dai (2012), uniform biasing fields in cylindrical coordinates are assumed here to enable an exact analysis. The three-dimensional equations governing the small-amplitude non-axisymmetric waves in incompressible soft electroactive hollow cylinders under uniform biasing fields are simplified and decoupled by introducing three displacement potentials. An exact solution is then derived in terms of Bessel functions. Numerical examples are finally presented to show the effects of biasing fields and other parameters on the wave propagation behavior.

2. Basic formulations

2.1. Nonlinear theory of electroelasticity

Consider an incompressible continuous electroelastic body. We denote the undeformed, stress-free configuration by B_r, and its boundary by ∂B_r, with $\mathbf{N}$ being the outward unit normal. Any material particle, say X, is labeled by a position vector $\mathbf{X}$. Let B_t denote the corresponding deformed configuration with ∂B_t the boundary and $\mathbf{n}$ the outward unit normal. The deformation is described by the mapping $\mathbf{x} = \chi(\mathbf{X}, t)$ where $\boldsymbol{\chi}$ is a continuous and twice differentiable vector function. The deformation gradient is defined by $\mathbf{F} = \mathrm{Grad}\,\chi$ with the Cartesian components given by $F_{i\alpha} = \partial x_i/\partial X_\alpha$. $\mathbf{b} = \mathbf{F}\mathbf{F}^T$ and $\mathbf{c} = \mathbf{F}^T\mathbf{F}$ are the left and right Cauchy–Green tensors respectively. The relations between the infinitesimal undeformed surface element $\mathrm{d}A$ and volume element $\mathrm{d}V$ and those deformed ones are specified by $\mathbf{n}\mathrm{d}a = J\mathbf{F}^{-\mathrm{T}}\mathbf{N}\mathrm{d}A$ and $\mathrm{d}v = J\mathrm{d}V$ respectively, where $J = |\mathbf{F}|$ is the determinant of the deformation gradient $\mathbf{F}$, also known as the volume ratio. We have $J = 1$ for incompressible materials.

Under the 'quasi-electrostatic approximation', the appropriate specializations of Maxwell's equations in the absence of free body charges and currents are

$$\mathrm{Curl}\,\mathbf{E}_l = \mathbf{0},\ \mathrm{Div}\,\mathbf{D}_l = 0, \tag{1}$$

where $\mathbf{E}_l = \mathbf{F}^T\mathbf{E}$ and $\mathbf{D}_l = \mathbf{F}^{-1}\mathbf{D}$ are the Lagrangian counterparts of the electric field vector $\mathbf{E}$ and electric displacement vector $\mathbf{D}$, respectively. Curl and Div are the curl and divergence operators defined in B_r, while curl and div will be used for the corresponding operators in B_t. The superscript T denotes the matrix transpose. In the vacuum outside the material, the electric field vector $\mathbf{E}*$ and electric displacement vector $\mathbf{D}*$ are related by

$$\mathbf{D}^* = \varepsilon_0\mathbf{E}^*, \tag{2}$$

where the constant ε_0 is the permittivity of vacuum. Obviously, we have

$$\mathrm{curl}\,\mathbf{E}^* = \mathbf{0},\ \mathrm{div}\,\mathbf{D}^* = 0. \tag{3}$$

The Maxwell stress in the vacuum is defined by

$$\tau^* = \varepsilon_0\left[\mathbf{E}^*\otimes\mathbf{E}^* - \frac{1}{2}(\mathbf{E}^*\cdot\mathbf{E}^*)\mathbf{I}\right]. \tag{4}$$

In the absence of surface charges, the jump conditions across the boundary read as

$$(\mathbf{E} - \mathbf{E}^*)\times\mathbf{n} = \mathbf{0},\quad (\mathbf{D} - \mathbf{D}^*)\cdot\mathbf{n} = 0. \tag{5}$$

The equations of equilibrium, in the absence of body forces, are

$$\mathrm{Div}\,\mathbf{T} = \mathbf{0}, \tag{6}$$

where $\mathbf{T} = \mathbf{F}^{-1}\tau = \partial\Omega/\partial\mathbf{F} - p\mathbf{F}^{-1}$ is the nominal stress tensor, with $\boldsymbol{\tau}$ being the total Cauchy stress tensor, $\Omega(\mathbf{F}, \mathbf{D}_l)$ is an amended energy function defined per unit volume in the reference configuration, and p is a Lagrange multiplier associated with the incompressibility constraint. p is identified as a hydrostatic pressure in Holzapfel (2000) and Dorfmann and Ogden (2014).

The mechanical boundary condition is given by

$$\tau\mathbf{n} = \mathbf{t}_a + \mathbf{t}_e, \tag{7}$$

here $\mathbf{t}_a$ is the applied mechanical traction per unit area of ∂B_t, and $\mathbf{t}_e = \tau^*\mathbf{n}$ is the contribution to the traction due to the electric field exterior to the body. Note that $\mathbf{t}_e$ is an unknown quantity, to be determined from the governing equations and the jump conditions.

2.2. Linear theory for incremental field

Following the formulation of Dorfmann and Ogden (2010b), we superimpose an incremental deformation $\dot{\mathbf{x}}(\mathbf{X}, t)$ along with an increment in the electric displacement $\dot{\mathbf{D}}_l$ upon the deformed configuration. The superposed dot is used in this paper to denote incremental quantities. The incremental forms of the governing Eqs. (1) and (6) are

$$\mathrm{curl}\,\dot{\mathbf{E}}_{l0} = \mathbf{0},\quad \mathrm{div}\,\dot{\mathbf{D}}_{l0} = 0, \tag{8}$$

$$\mathrm{div}\,\dot{\mathbf{T}}_0 = \rho\mathbf{u}_{,tt}, \tag{9}$$

where $\mathbf{u}(\mathbf{x}, t) = \mathbf{u}(\chi(\mathbf{X}, t), t) = \dot{\mathbf{x}}(\mathbf{X}, t)$ should be noticed, $\dot{\mathbf{T}}_0 = \mathbf{F}\dot{\mathbf{T}}$, $\dot{\mathbf{E}}_{l0} = \mathbf{F}^{-\mathrm{T}}\dot{\mathbf{E}}_l$, $\dot{\mathbf{D}}_{l0} = \mathbf{F}\dot{\mathbf{D}}_l$ are the 'push forward' versions of $\dot{\mathbf{T}}$, $\dot{\mathbf{E}}_l$, $\dot{\mathbf{D}}_l$ respectively. The linear incremental constitutive equations for an isotropic electroactive material are

$$\dot{\mathbf{T}}_0 = \mathbf{A}_0\mathbf{H} + \boldsymbol{\Gamma}_0\dot{\mathbf{D}}_{l0} + p\mathbf{H} - \dot{p}\mathbf{I},\quad \dot{\mathbf{E}}_{l0} = \boldsymbol{\Gamma}_0^T\mathbf{H} + \mathbf{K}_0\dot{\mathbf{D}}_{l0}, \tag{10}$$

where $\mathbf{H} = \mathrm{grad}\,\mathbf{u}$ is the displacement gradient. The components of the instantaneous electroelastic moduli tensors in Eq. (10) are

$$A_{0piqj} = F_{p\alpha}F_{q\beta}A_{\alpha i\beta j} = A_{0qjpi},\ \Gamma_{0piq} = F_{p\alpha}F_{\beta q}^{-1}\Gamma_{\alpha i\beta} = \Gamma_{0ipq},$$
$$K_{0ij} = F_{\alpha i}^{-1}F_{\beta j}^{-1}K_{\alpha\beta} = K_{0ji}, \tag{11}$$

with

$$A_{\alpha i\beta j} = \frac{\partial^2\Omega}{\partial F_{i\alpha}\partial F_{j\beta}},\quad \Gamma_{\alpha i\beta} = \frac{\partial^2\Omega}{\partial F_{i\alpha}\partial D_{l\beta}},\quad K_{\alpha\beta} = \frac{\partial^2\Omega}{\partial D_{l\alpha}\partial D_{l\beta}}. \tag{12}$$

Obviously, they depend on the applied biasing fields. Thus, the biasing fields can be a useful means to adjust the instantaneous material properties, which in turn have a profound effect on the incremental fields.

Similarly, the incremental forms of Maxwell's equations outside the material are

$$\mathrm{curl}\,\dot{\mathbf{E}}^* = \mathbf{0},\quad \mathrm{div}\,\dot{\mathbf{D}}^* = 0. \tag{13}$$

The incremental fields $\dot{\mathbf{E}}^*$ and $\dot{\mathbf{D}}^*$ are related by $\dot{\mathbf{D}}^* = \varepsilon_0\dot{\mathbf{E}}^*$. Accordingly, the incremental forms of the boundary conditions (5) and (7) are

$$(\dot{\mathbf{E}}_{l0} - \dot{\mathbf{E}}^* - \mathbf{H}^{\mathrm{T}}\mathbf{E}^*)\times\mathbf{n} = \mathbf{0},\ (\dot{\mathbf{D}}_{l0} + \mathbf{H}\mathbf{D}^* - \dot{\mathbf{D}}^*)\cdot\mathbf{n} = 0, \tag{14}$$

$$\dot{\mathbf{T}}_0^{\mathrm{T}}\mathbf{n} = \dot{\mathbf{t}}_{A0} + \dot{\tau}^*\mathbf{n} - \tau^*\mathbf{H}^{\mathrm{T}}\mathbf{n}, \tag{15}$$

where $\dot{\mathbf{t}}_{A0}\mathrm{d}a = \dot{\mathbf{t}}_A\mathrm{d}A$, with $\mathbf{t}_A$ being the applied mechanical traction per unit area of ∂B_r, and $\dot{\tau}^*$ is the incremental Maxwell stress given by

$$\dot{\tau}^* = \varepsilon_0[\dot{\mathbf{E}}^*\otimes\mathbf{E}^* + \mathbf{E}^*\otimes\dot{\mathbf{E}}^* - (\mathbf{E}^*\cdot\dot{\mathbf{E}}^*)\mathbf{I}]. \tag{16}$$

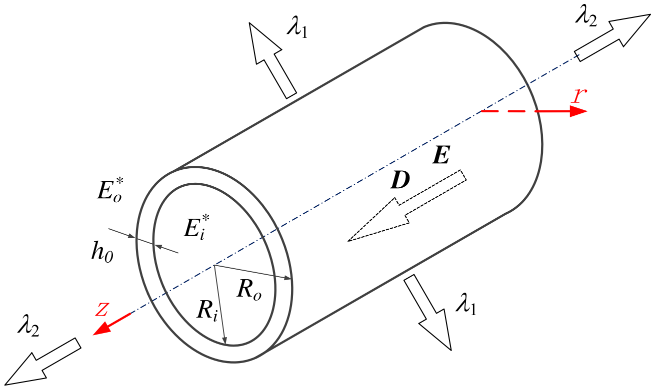

Fig. 1. An electroactive hollow cylinder under uniform electric and mechanical biasing fields.

The incremental incompressibility constraint relation is

$$\operatorname{div}\mathbf{u} = 0. \tag{17}$$

3. Governing equations with biasing fields

3.1. Uniform biasing fields and instantaneous material properties

Consider an infinitely long hollow cylinder made of an incompressible electroactive material. Its initial inner and outer radii are R_i and R_o, which become r_i and r_o after deformation, respectively. We now use cylindrical coordinate systems (R, Θ, Z) and (r, θ, z) to describe the undeformed and deformed configurations, respectively. To simplify the problem and facilitate the exact solution, we consider a uniform (pre-) deformation in the cylindrical coordinate system, as shown in Fig. 1. Thus, we have

$$r = \lambda_1 R, \quad \theta = \Theta, \quad z = \lambda_2 Z, \tag{18}$$

where λ_1 $(= r_i/R_i = r_o/R_o)$ and λ_2 are positive constants, specifying the stretches in the radial and axial directions, respectively. The biasing electric displacement vector $\mathbf{D}$ is uniform and taken to be aligned with the axial direction. Then, we have

$$\mathbf{F} = \begin{bmatrix} \frac{\partial r}{\partial R} & \frac{\partial r}{R\partial\Theta} & \frac{\partial r}{\partial Z} \\ \frac{r\partial\theta}{\partial R} & \frac{r\partial\theta}{R\partial\Theta} & \frac{r\partial\theta}{\partial Z} \\ \frac{\partial z}{\partial R} & \frac{\partial z}{R\partial\Theta} & \frac{\partial z}{\partial Z} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_1 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix},$$
$$\mathbf{b} = \mathbf{c} = \begin{bmatrix} \lambda_1^2 & 0 & 0 \\ 0 & \lambda_1^2 & 0 \\ 0 & 0 & \lambda_2^2 \end{bmatrix}, \quad \mathbf{D} = \begin{Bmatrix} 0 \\ 0 \\ D_z \end{Bmatrix}. \tag{19}$$

The incompressibility of the material requires

$$\lambda_2 = \lambda_1^{-2}. \tag{20}$$

According to Chen and Dai (2012), the non-zero components of the total stress tensor and electric field vector in the hollow cylinder ($r_i \le r \le r_o$) are

$$\tau_{rr} = \tau_{\theta\theta} = 2\Omega_1\lambda_1^2 + 2\Omega_2\left(I_1\lambda_1^2 - \lambda_1^4\right) - p,$$
$$\tau_{zz} = 2\Omega_1\lambda_2^2 + 2\Omega_2\left(I_1\lambda_2^2 - \lambda_2^4\right) - p + 2\Omega_5 D_z^2 + 4\Omega_6\lambda_2^2 D_z^2,$$
$$E_z = 2\left(\Omega_4\lambda_2^{-2} + \Omega_5 + \Omega_6\lambda_2^2\right)D_z, \tag{21}$$

where $\Omega_m = \partial\Omega/\partial I_m$, and $I_1 = \operatorname{tr}\mathbf{c}$, $I_2 = \frac{1}{2}[(\operatorname{tr}\mathbf{c})^2 - \operatorname{tr}(\mathbf{c}^2)]$, $I_3 = \det\mathbf{c} \equiv 1$, $I_4 = \mathbf{D}_l \cdot \mathbf{D}_l$, $I_5 = \mathbf{D}_l \cdot (\mathbf{c}\mathbf{D}_l)$, $I_6 = \mathbf{D}_l \cdot (\mathbf{c}^2\mathbf{D}_l)$ are the six independent invariants (Dorfmann and Ogden, 2010a; Chen and Dai, 2012). For the current problem, $I_1 = 2\lambda_1^2 + \lambda_2^2$ is the only invariant to be involved. We can deduce the electric displacement component D_z from Eq. (21)$_3$ if the component of electric field E_z is *a priori* known:

$$D_z = \frac{E_z}{2\left(\Omega_4\lambda_2^{-2} + \Omega_5 + \Omega_6\lambda_2^2\right)}. \tag{22}$$

The jump conditions (5) at $r = r_i$, where $\mathbf{n}_i = (-1, 0, 0)$ and those at $r = r_o$, where $\mathbf{n}_o = (1, 0, 0)$ become $E_z^* = E_z, E_\theta^* = D_r^* = 0$, from which and in view of Eq. (4), we have for the exterior electric fields ($r < r_i$ or $r > r_o$)

$$E_{oz}^* = E_{iz}^* = E_z = 2\left(\Omega_4\lambda_2^{-2} + \Omega_5 + \Omega_6\lambda_2^2\right)D_z,$$
$$D_{oz}^* = D_{iz}^* = 2\varepsilon_0\left(\Omega_4\lambda_2^{-2} + \Omega_5 + \Omega_6\lambda_2^2\right)D_z,$$
$$\tau_{orr}^* = \tau_{o\theta\theta}^* = -\tau_{ozz}^* = \tau_{irr}^* = \tau_{i\theta\theta}^* = -\tau_{izz}^*$$
$$= -2\varepsilon_0\left(\Omega_4\lambda_2^{-2} + \Omega_5 + \Omega_6\lambda_2^2\right)^2 D_z^2, \tag{23}$$

where the first subscript o or i indicates the outer or inner electric field.

It is apparent that, different from Shmuel and deBotton (2013), the stress and electric fields in the hollow cylinder and vacuum outside the material are all uniform. In this case, the balance laws (3) and (6) are satisfied automatically. In general $\tau_{rr} \neq \tau_{orr}^*$ and/or $\tau_{rr} \neq \tau_{irr}^*$, and additional mechanical tractions may be required to act on the boundaries of the hollow cylinder in order to satisfy the boundary conditions.

The instantaneous electroelastic moduli tensors A_{0ijkl}, K_{0ij}, Γ_{0ijk} can be evaluated according to Eq. (10), of which the nonzero components are given in Appendix A for easy reference.

3.2. Boundary value problem of the incremental fields

Consider a small-amplitude non-axisymmetric wave motion superimposed on the underlying deformed configuration of the hollow cylinder as prescribed in the previous subsection. Thus, we have

$$\mathbf{u} = u_r(r, \theta, z, t)\mathbf{e}_r + u_\theta(r, \theta, z, t)\mathbf{e}_\theta + u_z(r, \theta, z, t)\mathbf{e}_z,$$
$$\dot{\mathbf{D}}_{l0} = \dot{D}_{l0r}(r, \theta, z, t)\mathbf{e}_r + \dot{D}_{l0\theta}(r, \theta, z, t)\mathbf{e}_\theta + \dot{D}_{l0z}(r, \theta, z, t)\mathbf{e}_z,$$
$$\dot{p} = \dot{p}(r, \theta, z, t). \tag{24}$$

Accordingly, the displacement gradient is calculated to be

$$H = \begin{bmatrix} \frac{\partial u_r}{\partial r} & \frac{1}{r}\left(\frac{\partial u_r}{\partial\theta} - u_\theta\right) & \frac{\partial u_r}{\partial z} \\ \frac{\partial u_\theta}{\partial r} & \frac{1}{r}\left(\frac{\partial u_\theta}{\partial\theta} + u_r\right) & \frac{\partial u_\theta}{\partial z} \\ \frac{\partial u_z}{\partial r} & \frac{1}{r}\frac{\partial u_z}{\partial\theta} & \frac{\partial u_z}{\partial z} \end{bmatrix}. \tag{25}$$

The non-zero components of the incremental stress and electric fields are obtained from Eq. (10) as

$$\dot{T}_{0rr} = (A_{01111} + p)\frac{\partial u_r}{\partial r} + A_{01122}\frac{1}{r}\left(\frac{\partial u_\theta}{\partial\theta} + u_r\right)$$
$$+ A_{01133}\frac{\partial u_z}{\partial z} + \Gamma_{0113}\dot{D}_{l0z} - \dot{p},$$
$$\dot{T}_{0\theta\theta} = A_{01122}\frac{\partial u_r}{\partial r} + (A_{01111} + p)\frac{1}{r}\left(\frac{\partial u_\theta}{\partial\theta} + u_r\right)$$
$$+ A_{01133}\frac{\partial u_z}{\partial z} + \Gamma_{0113}\dot{D}_{l0z} - \dot{p},$$
$$\dot{T}_{0zz} = A_{01133}\frac{\partial u_r}{\partial r} + A_{01133}\frac{1}{r}\left(\frac{\partial u_\theta}{\partial\theta} + u_r\right) + (A_{03333} + p)\frac{\partial u_z}{\partial z}$$
$$+ \Gamma_{0333}\dot{D}_{l0z} - \dot{p},$$
$$\dot{T}_{0r\theta} = A_{01212}\frac{\partial u_\theta}{\partial r} + (A_{01221} + p)\frac{1}{r}\left(\frac{\partial u_r}{\partial\theta} - u_\theta\right),$$

$$\dot{T}_{0rz} = (A_{01331} + p)\frac{\partial u_r}{\partial z} + A_{01313}\frac{\partial u_z}{\partial r} + \Gamma_{0131}\dot{D}_{l0r},$$
$$\dot{T}_{0\theta r} = A_{01212}\frac{1}{r}\left(\frac{\partial u_r}{\partial \theta} - u_\theta\right) + (A_{01221} + p)\frac{\partial u_\theta}{\partial r},$$
$$\dot{T}_{0\theta z} = A_{01313}\frac{1}{r}\frac{\partial u_z}{\partial \theta} + (A_{01331} + p)\frac{\partial u_\theta}{\partial z} + \Gamma_{0131}\dot{D}_{l0\theta},$$
$$\dot{T}_{0zr} = A_{03131}\frac{\partial u_r}{\partial z} + (A_{01331} + p)\frac{\partial u_z}{\partial r} + \Gamma_{0131}\dot{D}_{l0r},$$
$$\dot{T}_{0z\theta} = A_{03131}\frac{\partial u_\theta}{\partial z} + (A_{01331} + p)\frac{1}{r}\frac{\partial u_z}{\partial \theta} + \Gamma_{0131}\dot{D}_{l0\theta},$$
$$\dot{E}_{l0r} = \Gamma_{0131}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) + K_{011}\dot{D}_{l0r},$$
$$\dot{E}_{l0\theta} = \Gamma_{0131}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) + K_{011}\dot{D}_{l0\theta},$$
$$\dot{E}_{l0z} = \Gamma_{0113}\left[\frac{\partial u_r}{\partial r} + \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right)\right] + \Gamma_{0333}\frac{\partial u_z}{\partial z} + K_{033}\dot{D}_{l0z}. \quad (26)$$

Eqs. (8), (9) and (17) can be written in the following component form

$$\frac{1}{r}\frac{\partial \dot{E}_{l0z}}{\partial \theta} - \frac{\partial \dot{E}_{l0\theta}}{\partial z} = 0,$$
$$\frac{\partial \dot{E}_{l0r}}{\partial z} - \frac{\partial \dot{E}_{l0z}}{\partial r} = 0,\ \frac{\partial \dot{E}_{l0\theta}}{\partial r} + \frac{\dot{E}_{l0\theta}}{r} - \frac{1}{r}\frac{\partial \dot{E}_{l0r}}{\partial \theta} = 0, \quad (27)$$

$$\frac{\partial \dot{D}_{l0r}}{\partial r} + \frac{1}{r}\left(\frac{\partial \dot{D}_{l0\theta}}{\partial \theta} + \dot{D}_{l0r}\right) + \frac{\partial \dot{D}_{l0z}}{\partial z} = 0, \quad (28)$$

$$\frac{\partial \dot{T}_{0rr}}{\partial r} + \frac{1}{r}\frac{\partial \dot{T}_{0\theta r}}{\partial \theta} + \frac{\dot{T}_{0rr} - \dot{T}_{0\theta\theta}}{r} + \frac{\partial \dot{T}_{0zr}}{\partial z} = \rho\frac{\partial^2 u_r}{\partial t^2},$$
$$\frac{\partial \dot{T}_{0r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \dot{T}_{0\theta\theta}}{\partial \theta} + \frac{\dot{T}_{0\theta r} + \dot{T}_{0r\theta}}{r} + \frac{\partial \dot{T}_{0z\theta}}{\partial z} = \rho\frac{\partial^2 u_\theta}{\partial t^2},$$
$$\frac{\partial \dot{T}_{0rz}}{\partial r} + \frac{1}{r}\frac{\partial \dot{T}_{0\theta z}}{\partial \theta} + \frac{\partial \dot{T}_{0zz}}{\partial z} + \frac{\dot{T}_{0rz}}{r} = \rho\frac{\partial^2 u_z}{\partial t^2}, \quad (29)$$

$$\frac{\partial u_r}{\partial r} + \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right) + \frac{\partial u_z}{\partial z} = 0. \quad (30)$$

The non-zero components of the incremental Maxwell stress are derived from Eq. (16)

$$\dot{\tau}^*_{mrr} = \dot{\tau}^*_{m\theta\theta} = -\dot{\tau}^*_{mzz} = -\varepsilon_0 E^*_{mz}\dot{E}^*_{mz}$$
$$\dot{\tau}^*_{mrz} = \dot{\tau}^*_{mzr} = \varepsilon_0 E^*_{mz}\dot{E}^*_{mr}, \quad \dot{\tau}^*_{m\theta r} = 0 (m = o, i). \quad (31)$$

We assume that there are no applied incremental mechanical tractions on the two cylindrical surfaces, *i.e.*, $\dot{\mathbf{t}}_{A0} = \mathbf{0}$ $(r = r_o, r_i)$. Then, on the boundaries $\mathbf{n}_i = (-1, 0, 0)$ and $\mathbf{n}_o = (1, 0, 0)$, the electric boundary conditions (14) lead to

$$\dot{E}_{l0\theta} - \dot{E}^*_{m\theta} - E^*_{mz}\frac{1}{r}\frac{\partial u_z}{\partial \theta} = 0, \dot{E}_{l0z} - \dot{E}^*_{mz} - E^*_{mz}\frac{\partial u_z}{\partial z} = 0,$$
$$\dot{D}_{l0r} + D^*_{mz}\frac{\partial u_r}{\partial z} - \dot{D}^*_{mr} = 0 (r = r_m;\ \ m = o, i); \quad (32)$$

while the mechanical boundary conditions (15) become

$$\dot{T}_{0rr} = \dot{\tau}^*_{mrr} - \tau^*_{mrr}\frac{\partial u_r}{\partial r}, \quad \dot{T}_{0r\theta} = \dot{\tau}^*_{m\theta r} - \tau^*_{m\theta\theta}\frac{1}{r}\left(\frac{\partial u_r}{\partial \theta} - u_\theta\right),$$
$$\dot{T}_{0rz} = \dot{\tau}^*_{mzr} - \tau^*_{mzz}\frac{\partial u_r}{\partial z} (r = r_m;\ \ m = o, i). \quad (33)$$

Similarly, the electric field in vacuum exterior to the material should satisfy the following electric balance equations

$$\frac{1}{r}\frac{\partial \dot{E}^*_{mz}}{\partial \theta} - \frac{\partial \dot{E}^*_{m\theta}}{\partial z} = 0,$$
$$\frac{\partial \dot{E}^*_{mr}}{\partial z} - \frac{\partial \dot{E}^*_{mz}}{\partial r} = 0,\ \frac{\partial \dot{E}^*_{m\theta}}{\partial r} + \frac{\dot{E}^*_{m\theta}}{r} - \frac{1}{r}\frac{\partial \dot{E}^*_{mr}}{\partial \theta} = 0, \quad (34)$$

$$\frac{\partial \dot{D}^*_{mr}}{\partial r} + \frac{1}{r}\left(\frac{\partial \dot{D}^*_{m\theta}}{\partial \theta} + \dot{D}^*_{mr}\right) + \frac{\partial \dot{D}^*_{mz}}{\partial z} = 0, \quad (35)$$

where $\dot{D}^*_{mj} = \varepsilon_0\dot{E}^*_{mj}$ $(m = o, i;\ \ j = r, z)$.

4. Exact wave solution and dispersion equation

4.1. Simplification of equations

From Eqs. (27) and (34) we deduce the existence of electric potentials ϕ and ϕ^*_m such that

$$\dot{E}_{l0r} = -\frac{\partial \phi}{\partial r}, \quad \dot{E}_{l0\theta} = -\frac{1}{r}\frac{\partial \phi}{\partial \theta}, \quad \dot{E}_{l0z} = -\frac{\partial \phi}{\partial z}, \quad (36)$$

$$\dot{E}^*_{mr} = -\frac{\partial \phi^*_m}{\partial r}, \quad \dot{E}^*_{m\theta} = -\frac{1}{r}\frac{\partial \phi^*_m}{\partial \theta}, \quad \dot{E}^*_{mz} = -\frac{\partial \phi^*_m}{\partial z} (m = o, i). \quad (37)$$

In this way, Eqs. (27) and (34) are automatically satisfied. From Eq. (35), we know that ϕ^*_m should satisfy the Laplace's equation

$$\nabla^2\phi^*_m + \frac{\partial^2\phi^*_m}{\partial z^2} = 0 \qquad (m = o, i), \quad (38)$$

where $\nabla^2 = \partial^2/\partial r^2 + (1/r)\partial/\partial r + (1/r^2)\partial^2/\partial\theta^2$ is the two-dimensional Laplace operator. Simultaneously, the constitutive relations (26) can be rewritten in terms of the electric potential ϕ as

$$\dot{D}_{l0r} = e_{15}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) - \varepsilon_{11}\frac{\partial \phi}{\partial r},$$
$$\dot{D}_{l0\theta} = e_{15}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) - \varepsilon_{11}\frac{1}{r}\frac{\partial \phi}{\partial \theta},$$
$$\dot{D}_{l0z} = e_{31}\left[\frac{\partial u_r}{\partial r} + \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right)\right] + e_{33}\frac{\partial u_z}{\partial z} - \varepsilon_{33}\frac{\partial \phi}{\partial z},$$
$$\dot{T}_{0rr} = (c_{11} + p)\frac{\partial u_r}{\partial r} + c_{12}\frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right) + c_{13}\frac{\partial u_z}{\partial z} + e_{31}\frac{\partial \phi}{\partial z} - \dot{p},$$
$$\dot{T}_{0\theta\theta} = c_{12}\frac{\partial u_r}{\partial r} + (c_{11} + p)\frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right) + c_{13}\frac{\partial u_z}{\partial z} + e_{31}\frac{\partial \phi}{\partial z} - \dot{p},$$
$$\dot{T}_{0zz} = c_{13}\frac{\partial u_r}{\partial r} + c_{13}\frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right) + (c_{33} + p)\frac{\partial u_z}{\partial z} + e_{33}\frac{\partial \phi}{\partial z} - \dot{p},$$
$$\dot{T}_{0r\theta} = c_{14}\frac{\partial u_\theta}{\partial r} + (c_{15} + p)\frac{1}{r}\left(\frac{\partial u_r}{\partial \theta} - u_\theta\right),$$
$$\dot{T}_{0\theta r} = c_{14}\frac{1}{r}\left(\frac{\partial u_r}{\partial \theta} - u_\theta\right) + (c_{15} + p)\frac{\partial u_\theta}{\partial r},$$
$$\dot{T}_{0rz} = (c_{551} + p)\frac{\partial u_r}{\partial z} + c_{552}\frac{\partial u_z}{\partial r} + e_{15}\frac{\partial \phi}{\partial r},$$
$$\dot{T}_{0zr} = c_{553}\frac{\partial u_r}{\partial z} + (c_{551} + p)\frac{\partial u_z}{\partial r} + e_{15}\frac{\partial \phi}{\partial r},$$
$$\dot{T}_{0\theta z} = c_{552}\frac{1}{r}\frac{\partial u_z}{\partial \theta} + (c_{551} + p)\frac{\partial u_\theta}{\partial z} + e_{15}\frac{1}{r}\frac{\partial \phi}{\partial \theta},$$
$$\dot{T}_{0z\theta} = c_{553}\frac{\partial u_\theta}{\partial z} + (c_{551} + p)\frac{1}{r}\frac{\partial u_z}{\partial \theta} + e_{15}\frac{1}{r}\frac{\partial \phi}{\partial \theta}, \quad (39)$$

where

$$\varepsilon_{11} = 1/K_{011}, \quad \varepsilon_{33} = 1/K_{033}, \quad e_{15} = -\Gamma_{0131}\varepsilon_{11},$$
$$e_{31} = -\Gamma_{0113}\varepsilon_{33}, \quad e_{33} = -\Gamma_{0333}\varepsilon_{33}, \quad c_{11} = A_{01111} + \Gamma_{0113}e_{31},$$
$$c_{12} = A_{01122} + \Gamma_{0113}e_{31}, \quad c_{13} = A_{01133} + \Gamma_{0113}e_{33},$$
$$c_{33} = A_{03333} + \Gamma_{0333}e_{33}, \quad c_{551} = A_{01331} + \Gamma_{0131}e_{15},$$
$$c_{14} = A_{01212}, \quad c_{15} = A_{01221},$$
$$c_{552} = A_{01313} + \Gamma_{0131}e_{15}, \quad c_{553} = A_{03131} + \Gamma_{0131}e_{15}. \quad (40)$$

Substituting Eq. (39) into Eqs. (28) and (29), we obtain

$$\left[e_{15}\nabla^2+(e_{33}-e_{31}-e_{15})\frac{\partial^2}{\partial z^2}\right]u_z-\left(\varepsilon_{11}\nabla^2+\varepsilon_{33}\frac{\partial^2}{\partial z^2}\right)\phi=0$$

$$\left[c_{11}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)+c_{14}\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+c_{553}\frac{\partial^2}{\partial z^2}\right]u_r$$
$$+\left[(c_{15}+c_{12})\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}-(c_{11}+c_{14})\frac{1}{r^2}\frac{\partial}{\partial\theta}\right]u_\theta$$
$$+(c_{13}+c_{551})\frac{\partial^2 u_z}{\partial r\partial z}+(e_{31}+e_{15})\frac{\partial^2\phi}{\partial r\partial z}-\frac{\partial\dot{p}}{\partial r}=\rho\frac{\partial^2 u_r}{\partial t^2},$$

$$\left[(c_{15}+c_{12})\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}+(c_{11}+c_{14})\frac{1}{r^2}\frac{\partial}{\partial\theta}\right]u_r$$
$$+\left[c_{14}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)+c_{11}\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+c_{553}\frac{\partial^2}{\partial z^2}\right]u_\theta$$
$$+(c_{13}+c_{551})\frac{1}{r}\frac{\partial^2 u_z}{\partial\theta\partial z}+(e_{31}+e_{15})\frac{1}{r}\frac{\partial^2\phi}{\partial\theta\partial z}-\frac{1}{r}\frac{\partial\dot{p}}{\partial\theta}=\rho\frac{\partial^2 u_\theta}{\partial t^2},$$

$$(c_{551}+c_{13})\left(\frac{\partial^2}{\partial r\partial z}+\frac{1}{r}\frac{\partial}{\partial z}\right)u_r+(c_{13}+c_{551})\frac{1}{r}\frac{\partial^2 u_\theta}{\partial\theta\partial z}$$
$$+\left[c_{552}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)+c_{33}\frac{\partial^2}{\partial z^2}\right]u_z$$
$$+\left[e_{15}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)+e_{33}\frac{\partial^2}{\partial z^2}\right]\phi$$
$$-\frac{\partial\dot{p}}{\partial z}=\rho\frac{\partial^2 u_z}{\partial t^2}. \tag{41}$$

The above four equations along with the incompressibility constraint (30) govern the incremental motion of the electroactive hollow cylinder in terms of five unknown functions $u_r, u_\theta, u_z, \phi, \dot{p}$. It can be found that these equations are similar in form to those for the infinitesimal motion of a transversely isotropic elastic body (Ding et al., 2006). Thus, we may introduce the following three displacement functions (Ding et al., 2006)

$$u_r=\frac{1}{r}\frac{\partial\psi}{\partial\theta}-\frac{\partial G}{\partial r},\quad u_\theta=-\frac{\partial\psi}{\partial r}-\frac{1}{r}\frac{\partial G}{\partial\theta},\quad u_z=W. \tag{42}$$

By substituting Eq. (42) into Eqs. (41) and (30) and noticing the relation $c_{11}-c_{12}-c_{15}=c_{14}$, we can transform the original governing equations into

$$\left(c_{14}\nabla^2+c_{553}\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right)\psi=0,$$

$$\left[(c_{11}-c_{13}-c_{551})\nabla^2+c_{553}\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right]G$$
$$-(e_{31}+e_{15})\frac{\partial\phi}{\partial z}+\dot{p}=0,$$

$$\left[c_{552}\nabla^2+(c_{33}-c_{13}-c_{551})\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right]W$$
$$+\left(e_{15}\nabla^2+e_{33}\frac{\partial^2}{\partial z^2}\right)\phi-\frac{\partial\dot{p}}{\partial z}=0,$$

$$\left[e_{15}\nabla^2+(e_{33}-e_{31}-e_{15})\frac{\partial^2}{\partial z^2}\right]W-\left(\varepsilon_{11}\nabla^2+\varepsilon_{33}\frac{\partial^2}{\partial z^2}\right)\phi=0,$$
$$-\nabla^2 G+\frac{\partial W}{\partial z}=0. \tag{43}$$

It is seen that the resulting equations are much simpler than the original ones. Moreover, the function ψ is successfully decoupled from the other four unknown functions, which is also very beneficial to the understanding of the induced wave motion.

4.2. Exact non-axisymmetric wave solution

We assume the traveling wave solution in the following form

$$\psi=\bar{\psi}(r)\sin(n\theta)\cos(kz-\omega t),\quad G=\bar{G}(r)\cos(n\theta)\cos(kz-\omega t),$$
$$W=\bar{W}(r)\cos(n\theta)\sin(kz-\omega t),\quad \phi=\bar{\phi}(r)\cos(n\theta)\sin(kz-\omega t),$$
$$\dot{p}=\bar{p}(r)\cos(n\theta)\cos(kz-\omega t), \tag{44}$$

where ω is the angular frequency, and n and k are the circumferential and axial wave numbers respectively. It is noted that $n=0$ corresponds to axisymmetric waves (for which $\psi=0$ and $u_\theta=0$) that have been studied by Chen and Dai (2012). By substituting the above wave solution into Eq. (43) and making use of $(43)_5$, we obtain

$$\left(\Lambda+\alpha_4^2\right)\bar{\psi}=0, \tag{45}$$

$$[(c_{11}-c_{13}-c_{551})\Lambda-c_{553}k^2+\rho\omega^2]\bar{G}-(e_{31}+e_{15})k\bar{\phi}+\bar{p}=0,$$
$$[c_{552}\Lambda-(c_{33}-c_{13}-c_{551})k^2+\rho\omega^2]\bar{W}$$
$$+(e_{15}\Lambda-e_{33}k^2)\phi+k\bar{p}=0,$$
$$[e_{15}\Lambda-(e_{33}-e_{31}-e_{15})k^2]\bar{W}-(\varepsilon_{11}\Lambda-\varepsilon_{33}k^2)\bar{\phi}=0,$$
$$-\Lambda\bar{G}+k\bar{W}=0, \tag{46}$$

where $\Lambda=\mathrm{d}^2/\mathrm{d}r^2+(1/r)\mathrm{d}/\mathrm{d}r-n^2/r^2$, and $\alpha_4^2=(\rho\omega^2-c_{553}k^2)/c_{14}$ is the radial wave number of the shear waves that can be expressed by the function ψ only. It is easy to show that the dilation associated with the motion represented by ψ vanishes, indicating the motion is purely shear and elastic.

Eq. (45) is a Bessel's equation of order n, whose solution is

$$\bar{\psi}=A_4J_n(\alpha_4 r)+B_4Y_n(\alpha_4 r), \tag{47}$$

where $J_n(\cdot)$ and $Y_n(\cdot)$ are the Bessel functions of the first and second kinds of order n, respectively. Eq. (46) looks more complex, but its solution can be sought by assuming (Ding et al., 1997)

$$\begin{Bmatrix}\bar{G}\\\bar{W}\\\bar{\phi}\\\bar{p}\end{Bmatrix}=J_n(\alpha r)\begin{Bmatrix}C_1\\C_2\\C_3\\C_4\end{Bmatrix}+Y_n(\alpha r)\begin{Bmatrix}D_1\\D_2\\D_3\\D_4\end{Bmatrix}, \tag{48}$$

where α is the radial wave number for the coupled waves associated with the other four functions, and C_j and $D_j (j=1\sim 4)$ are constants to be determined. It is noted that for these coupled waves, the axial component of the rotational vector vanishes. Substituting the solution into Eq. (46) yields a set of linear equations about the 8 unknown constants. For non-trivial solutions to exist, the determinant of the matrix consisting of the coefficients of C_j and $D_j (j=1\sim 4)$ must vanish, giving rise to the following characteristic equation

$$\begin{vmatrix}g_{11}&0&g_{13}&1\\0&g_{22}&g_{23}&g_{24}\\0&g_{32}&g_{33}&0\\g_{41}&g_{42}&0&0\end{vmatrix}=0, \tag{49}$$

where

$$g_{11}=-(c_{11}-c_{13}-c_{551})\alpha^2+\rho\omega^2-c_{553}k^2,\quad g_{13}=-(e_{31}+e_{15})k,$$
$$g_{22}=-c_{552}\alpha^2+\rho\omega^2-(c_{33}-c_{13}-c_{551})k^2,\quad g_{23}=-e_{15}\alpha^2-e_{33}k^2,$$
$$g_{32}=e_{15}\alpha^2+(e_{33}-e_{31}-e_{15})k^2,\quad g_{33}=-\varepsilon_{11}\alpha^2-\varepsilon_{33}k^2,$$
$$g_{41}=\alpha^2,\quad g_{42}=g_{24}=k. \tag{50}$$

The elements of the determinant in Eq. (49) are functions of k, ω and α. Once k and ω are given, we can obtain from Eq. (49) three different α's with $\mathrm{Re}[\alpha_j]>0$ or $\mathrm{Re}[\alpha_j]=0$ and $\mathrm{Im}[\alpha_j]>0$. Thus, the complete coupled wave solution can be written as

$$\begin{Bmatrix}\bar{G}\\\bar{W}\\\bar{\phi}\\\bar{p}\end{Bmatrix}=\sum_{i=1}^{3}\begin{Bmatrix}1\\\beta_{1j}\\\beta_{2j}\\\beta_{3j}\end{Bmatrix}[A_jJ_n(\alpha_j r)+B_jY_n(\alpha_j r)], \tag{51}$$

where $\beta_{nj}(n = 1, 2, 3)$ are ratios between the constants and can be obtained as

$$\beta_{1j} = -\frac{\alpha_j^2}{k}, \quad \beta_{2j} = \frac{e_{15}\alpha_j^2 + (e_{33} - e_{31} - e_{15})k^2}{\varepsilon_{11}\alpha_j^2 + \varepsilon_{33}k^2}\beta_{1j},$$

$$\beta_{3j} = -\frac{1}{k}\{[\rho\omega^2 - c_{552}\alpha_j^2 - (c_{33} - c_{13} - c_{551})k^2]\beta_{1j} - (e_{15}\alpha_j^2 + e_{33}k^2)\beta_{2j}\}(j = 1, 2, 3). \tag{52}$$

It is noted that we do not distinguish above different cases regarding the value of α (*i.e.*, real, imaginary or complex) as Mirsky (1965) did. This is simply because the present computational software such as Mathematica or MatLab can perform the complex numerical calculation directly and accurately.

Similarly, we assume the solution of $\phi_m^*(m = o, i)$ in the form

$$\phi_m^* = \bar{\phi}_m^* \cos(n\theta)\sin(kz - \omega t). \tag{53}$$

Substituting into Eq. (38) leads to a modified Bessel's equation

$$(\Lambda - k^2)\bar{\phi}_m^* = 0. \tag{54}$$

Its solution is

$$\bar{\phi}_m^* = A_m^* Z_n(kr), \tag{55}$$

where A_m^* $(m = o, i)$ are constants to be determined. Note that for $m = o$, we have $Z_n(\cdot) = K_n(\cdot)$ which is the modified Bessel function of the second kind of order n; while for $m = i$, $Z_n(\cdot) = I_n(\cdot)$ is the modified Bessel function of the first kind of order n.

With the aid of Eqs. (44) and (55), the stresses, electric displacements and electric fields in the hollow cylinder and vacuum exterior to the material can be obtained. These expressions are given in Appendix B for reference.

4.3. Dispersion relation

Substituting the stresses, electric displacements and electric fields into the boundary conditions (32) and (33), noting that $(32)_1$ is equivalent to $(32)_2$, we obtain for non-trivial solutions to exist

$$|d_{ij}| = 0(i, j = 1 \sim 10), \tag{56}$$

which is the dispersion equation. The elements of the determinant, d_{ij}, are given in Appendix C.

If we consider a solid cylinder, the solution should be kept finite at $r = 0$. Then $B_j = 0$ $(j = 1 \sim 4)$, and $\phi_i^* = 0$. The satisfaction of the boundary conditions at the outer surface leads to

$$\begin{vmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{19} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{29} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{39} \\ d_{41} & d_{42} & d_{43} & d_{44} & d_{49} \\ d_{51} & d_{52} & d_{53} & d_{54} & d_{59} \end{vmatrix} = 0. \tag{57}$$

5. Numerical examples and discussion

5.1. Material model

For numerical illustration, we adopt the modified neo-Hookean model (Dorfmann and Ogden, 2010a) with the following free energy function

$$\Omega = \frac{1}{2}\mu(I_1 - 3) + \frac{1}{\varepsilon_0}(\gamma_1 I_4 + \gamma_2 I_5). \tag{58}$$

The first term is the energy function for the classical neo-Hookean elastic material, with μ being the shear modulus of the material. The second term accounts for the interaction between the finite deformation and the electric field, and γ_1, γ_2 are two dimensionless electroelastic coupling parameters. If $\gamma_1 = 0$ the energy function corresponds to the so-called ideal dielectric elastomer (Zhao and Suo, 2007).

We obtain from Eqs. $(21)_{1,2}$, that

$$\tau_{rr} = \tau_{\theta\theta} = \mu\lambda_1^2 - p, \quad \tau_{zz} = \mu\lambda_2^2 - p + 2\varepsilon_0^{-1}\gamma_2 D_z^2, \tag{59}$$

which shows that the parameter γ_1 does not affect the total stress, which can be immediately seen from the energy function (58) because I_4 does not depend on $\mathbf{F}$, and γ_2 is a measure of how the stresses in the material are influenced by the electric field. If $\gamma_2 = 0$, the stress is independent of the electric field.

The Maxwell stresses and electric fields outside the body can be obtained from Eq. (23)

$$E_{oz}^* = E_{iz}^* = E_z = 2\varepsilon_0^{-1}\left(\gamma_1\lambda_2^{-2} + \gamma_2\right)D_z,$$
$$D_{oz}^* = D_{iz}^* = 2\left(\gamma_1\lambda_2^{-2} + \gamma_2\right)D_z,$$
$$\tau_{mrr}^* = \tau_{m\theta\theta}^* = -\tau_{mzz}^* = -2\varepsilon_0^{-1}\left(\gamma_1\lambda_2^{-2} + \gamma_2\right)^2 D_z^2(m = o, i). \tag{60}$$

It is apparent that the parameter γ_1 characterizes the electric response of the material in terms of the deformation.

The material parameters defined in Eq. (40) may be evaluated in respect of the model (58) as

$$c_{11} = c_{14} = \mu\lambda_1^2, \quad c_{12} = c_{13} = c_{15} = 0,$$
$$c_{33} = \mu\lambda_1^{-4} + 2\varepsilon_0^{-1}\gamma_2 D_z^2 - \frac{8\varepsilon_0^{-1}\gamma_2^2 D_z^2}{\gamma_1\lambda_1^4 + \gamma_2},$$
$$c_{551} = -\frac{2\varepsilon_0^{-1}\gamma_2^2 D_z^2}{\gamma_1\lambda_1^{-2} + \gamma_2}, \quad c_{552} = \mu\lambda_1^2 + c_{551},$$
$$c_{553} = \mu\lambda_1^{-4} + 2\varepsilon_0^{-1}\gamma_2 D_z^2 + c_{551},$$
$$\varepsilon_{11} = \frac{\varepsilon_0}{2\left(\gamma_1\lambda_1^{-2} + \gamma_2\right)}, \quad \varepsilon_{33} = \frac{\varepsilon_0}{2\left(\gamma_1\lambda_1^4 + \gamma_2\right)},$$
$$e_{15} = -\frac{\gamma_2 D_z}{\gamma_1\lambda_1^{-2} + \gamma_2}, \quad e_{31} = 0, \quad e_{33} = -\frac{2\gamma_2 D_z}{\gamma_1\lambda_1^4 + \gamma_2}. \tag{61}$$

One may notice that, with the chosen constitutive law, $c_{12} = c_{13} = 0$, which looks a little strange at first glance since they generally correspond to the Poisson effects in a usual elastic material. However, it does not indicate the disappearing of the Poisson effect (with regard to the incremental motion), because of the constraint of material incompressibility for incompressible materials.

We further assume here that there is no applied mechanical traction on the two cylindrical surfaces. Then p can be determined as $p = \mu\lambda_1^2 - \tau_{orr}^* = \mu\lambda_1^2 - \tau_{irr}^*$.

The numerical results will be displayed in terms of the following non-dimensional quantities

$$\varpi = \frac{\omega h_0}{c_T}, \quad \kappa = kh_0, \quad \delta = D_z/\sqrt{\mu\varepsilon_0}, \tag{62}$$

where $h_0 \equiv R_o - R_i$ is the initial thickness of the hollow cylinder, and

$$c_T = \sqrt{\frac{\mu}{\rho}} \tag{63}$$

is the transverse wave velocity in the undeformed isotropic electroactive material.

5.2. $n = 0$

We first examine the axisymmetric waves for which $n = 0$. In this case, the displacements are independent of θ and the dispersion Eq. (56) can be represented as the product of two determinants

$$S_1 S_2 = 0, \tag{64}$$

where

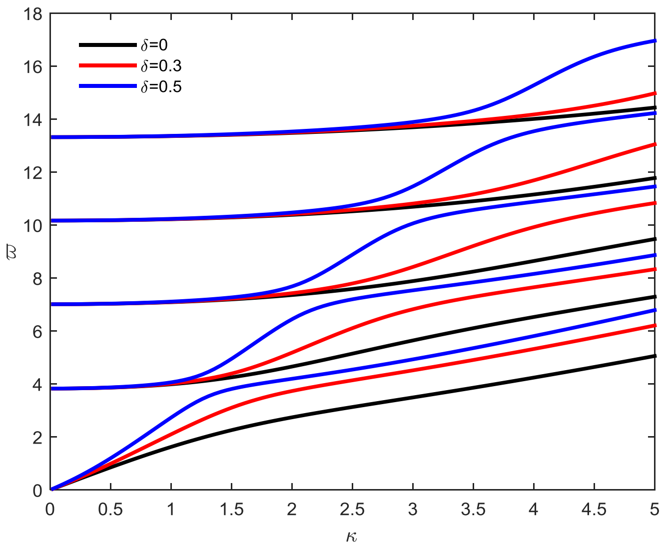

Fig. 2. Frequency spectrum for the first five axisymmetric torsionless wave modes in a solid cylinder.

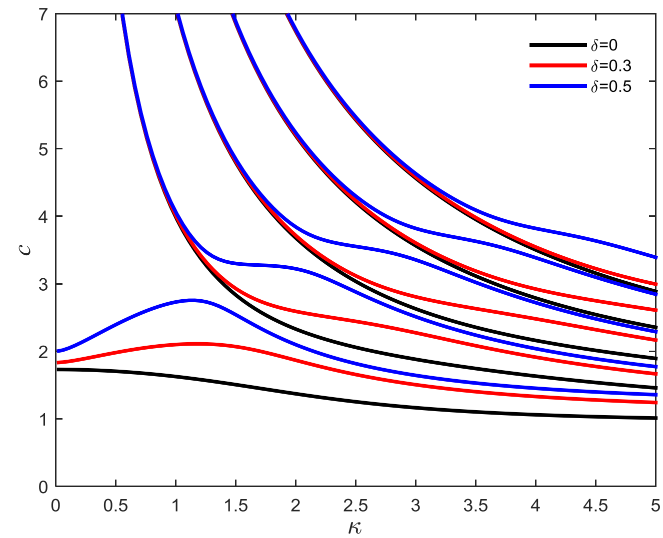

Fig. 3. Phase velocity curves for the first five axisymmetric torsionless wave modes in a solid cylinder.

$$S_1 = \begin{vmatrix} d_{11} & d_{12} & d_{13} & d_{15} & d_{16} & d_{17} & d_{19} & d_{110} \\ d_{31} & d_{32} & d_{33} & d_{35} & d_{36} & d_{37} & d_{39} & d_{310} \\ d_{41} & d_{42} & d_{43} & d_{45} & d_{46} & d_{47} & d_{49} & d_{410} \\ d_{51} & d_{52} & d_{53} & d_{55} & d_{56} & d_{57} & d_{59} & d_{510} \\ d_{61} & d_{62} & d_{63} & d_{65} & d_{66} & d_{67} & d_{69} & d_{610} \\ d_{81} & d_{82} & d_{83} & d_{85} & d_{86} & d_{87} & d_{89} & d_{810} \\ d_{91} & d_{92} & d_{93} & d_{95} & d_{96} & d_{97} & d_{99} & d_{910} \\ d_{101} & d_{102} & d_{103} & d_{105} & d_{106} & d_{107} & d_{109} & d_{1010} \end{vmatrix},$$

$$S_2 = \begin{vmatrix} d_{24} & d_{28} \\ d_{74} & d_{78} \end{vmatrix}. \tag{65}$$

The dispersion equation

$$S_1 = 0 \tag{66}$$

represents the θ-independent coupled motion with both radial and axial displacements but without the circumferential displacement (*i.e.*, $u_\theta = 0$). On the other hand,

$$S_2 = 0 \tag{67}$$

corresponds to the purely torsional motion with the only displacement component u_θ that is also independent of θ. It should be noted that the axisymmetric torsional wave solution may be obtained by exchanging $\cos(n\theta)$ and $\sin(n\theta)$ in Eq. (44) and then letting $n = 0$.

When a solid cylinder is considered, the dispersion Eq. (67) degenerates to

$$S_2 = d_{24} = 0, \tag{68}$$

which can be rewritten as

$$\lambda_1 \alpha_4 b J_0(\alpha_4 b) - 2J_1(\alpha_4 b) = 0. \tag{69}$$

Furthermore, if we omit the effect of biasing fields ($\lambda_1 = 1, D_z = 0$), we obtain

$$\alpha_4^2 = \frac{\rho\omega^2 - c_{553}k^2}{c_{14}} = \frac{\omega^2}{c_L^2} - k^2. \tag{70}$$

This is the classical result for torsional waves in elastic cylinders (Achenbach, 1984).

Figs. 2 and 3 display respectively the dimensionless frequency ϖ and phase velocity $c = \varpi/\kappa$ as functions of the dimensionless wave number κ for the first five axisymmetric torsionless wave modes in the soft electroactive solid cylinder with axial constraint $\lambda_2 = 1$ subjected to different biasing electric fields. The dimensionless electroelastic coupling parameters are taken to be $\gamma_1 = 1, \gamma_2 = 3$. It can be seen that all the modes are dispersive and the fundamental mode is the only one with no cut-off frequency. The case $\delta = 0$ corresponds to the particular case for which there is no biasing electric field. In this case, we actually obtain the dispersion curves in the two figures for the family of axisymmetric torsionless wave modes in an elastic solid cylinder. However, compared with Royer and Dieulesaint (2000), we find, due to material incompressibility, the branches associated with dilatational motion are not included here. In the limit as $\kappa \to 0$, the phase velocity of the fundamental mode with the lowest frequency becomes equal to $c = \sqrt{3}$ for $\delta = 0$. This coincides with the classical result for an elastic solid cylinder (Mirsky and Herrmann, 1958). If we choose the dimensionless material constants to be $\gamma_1 = 1.5, \gamma_2 = 2$, we obtain the same results, indicating that in the absence of electric field ($\delta = 0$), the electroelastic coupling parameters have no influence on the wave propagation characteristics. This actually can be observed directly from the analytical results in Eq. (61). We also note that, although the instantaneous dielectric constants ε_{11} and ε_{33} are still affected by the electroelastic coupling parameters, we have in this case $e_{ij} = 0$. Thus, as can be seen from Eq. (46), the elastic and electric fields are decoupled from each other. The influence of the biasing electric field can be seen from Fig. 3 clearly: The phase velocity increases with the magnitude of the biasing field for both short and long waves. Moreover, there is a region that for long waves (with a small value of κ) the biasing electric field has a relatively small effect on the dispersion characteristics, while for short waves, the biasing electric field changes shape of the dispersion curve quite obviously. This region starts later for higher modes. In particular, the velocity curve of the fundamental mode under a relatively large biasing electric field becomes nonmonotonic — The phase velocity first increases with the wave number, then arrives at a maximum, and finally decreases with the wave number. A similar observation can be obtained from the results presented in Shmuel et al. (2012). Such a nonmonotonic variation of dispersion curves should be a result of complex wave interaction in terms of geometric boundaries and material properties.

Of special interest is the fundamental mode which is the most important from a practical viewpoint since the vibration mode with the lowest frequency will be easily excited by the environmental disturbance which is usually with a low characteristic frequency. We therefore will present the results for the fundamental mode only in the following. Figs. 4 and 5 give the results for axisymmetric torsional waves. We can see from Fig. 4 that there is a cutoff frequency ϖ_c (≈ 5.13) at $\kappa = 0$. That is, only for frequency $\varpi > \varpi_c$, there will be

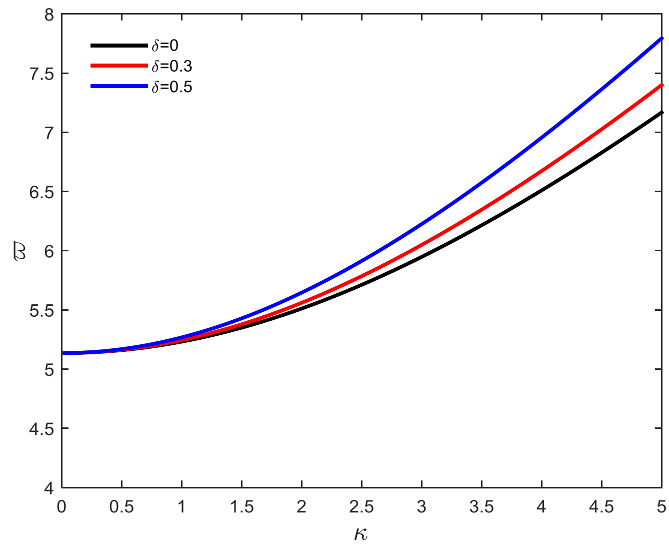

Fig. 4. Lowest frequency versus wave number for axisymmetric torsional waves in a solid cylinder.

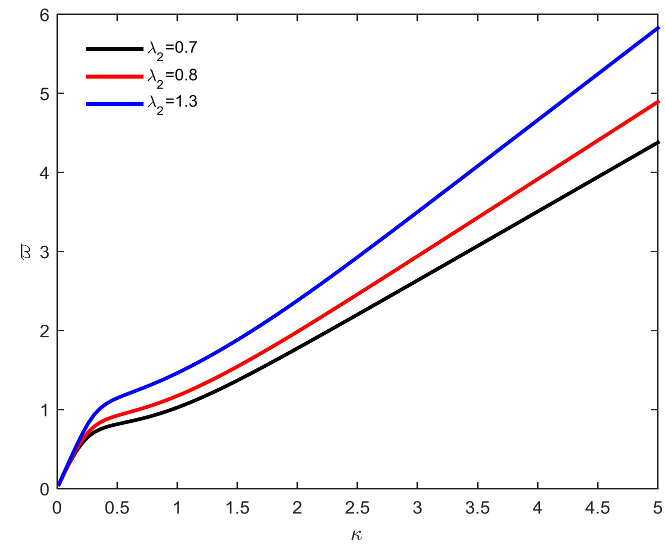

Fig. 6. Lowest frequency versus wave number for flexural waves in a hollow cylinder ($\eta = 1/3$).

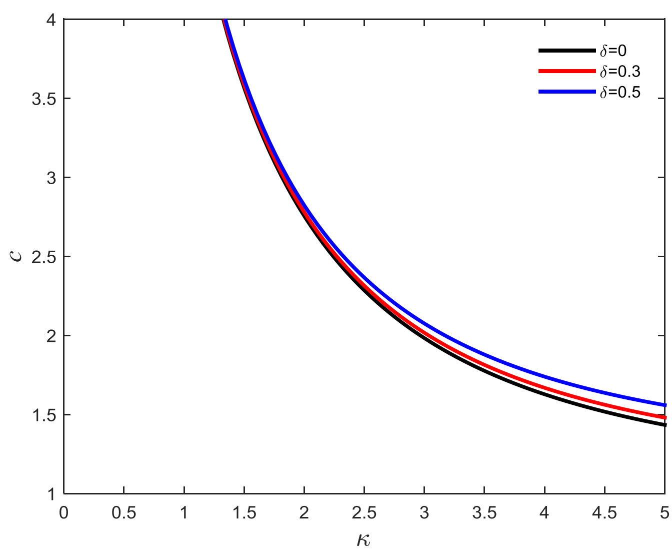

Fig. 5. Phase velocity versus wave number for axisymmetric torsional waves in a solid cylinder.

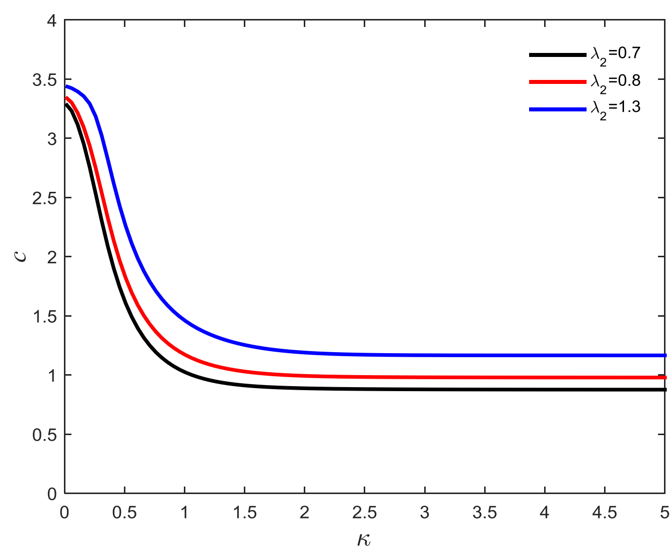

Fig. 7. Phase velocity versus wave number for flexural waves in a hollow cylinder ($\eta = 1/3$).

a torsional wave propagating in a solid cylinder. While for $\varpi < \varpi_c$, the wavenumber becomes imaginary, which corresponds to a non-propagating disturbance. The special case $\varpi = \varpi_c$ represents the transition from propagation to non-propagation. In this case, the solid cylinder is in uniform vibration since $\kappa = 0$ indicates the wavelength is infinitely long.

Since the biasing electric field leads to an obvious electroelastic coupling, making the electroactive material behave like piezoelectric material, the so-called piezoelectric stiffening effect (Yang, 2004) can be clearly identified from these figures.

5.3. n ≠ 0

Of the flexural waves with the components of displacement dependent on θ the family of waves specified by $n = 1$, known as the fundamental flexural waves is the most important. Figs. 6 and 7 show the curves of fundamental frequency and velocity versus wave number for the fundamental waves in the hollow cylinder subjected to different levels of pre-stretch. Here we fix the normalized electroelastic coupling parameters as $\gamma_1 = 1$, $\gamma_2 = 3$, the biasing electric displacement as $\delta = 0.5$, and the radius ratio of the cylindrical shell as $\eta = R_i/R_o = 1/3$. The influence of the biasing mechanical field is evident: As the pre-stretch increases a rise in the frequency and velocity is exhibited. However, for a finite structure, the pre-stretch does not always improve the global rigidity of the structure (and hence the frequency) since a competition exists between the increasing size of the structure and the increasing elastic constants of the material (Wang et al., 2013).

To examine the influence of the geometry of the hollow cylinder, the frequency and phase velocity curves for different radius ratios are compared in Figs. 8 and 9, with $n = 1$, $\lambda_2 = 0.8$, $\gamma_1 = 1$, $\gamma_2 = 3$, $\delta = 0.5$. The results show that the geometry of the hollow cylinder has a significant effect on the wave propagation characteristic for small values of κ ($\kappa < 1.5$). However, as $\kappa \to \infty$, the phase velocity always approaches that of the Rayleigh surface wave in an electroactive half-space, as expected. Therefore, the varying of the radius ratio can hardly affect the frequency/phase velocity at a high wave number.

The external electric field in vacuum is usually ignored in literature when traditional piezoelectric materials and structures are considered. For soft electroactive materials, it may play a critical role.

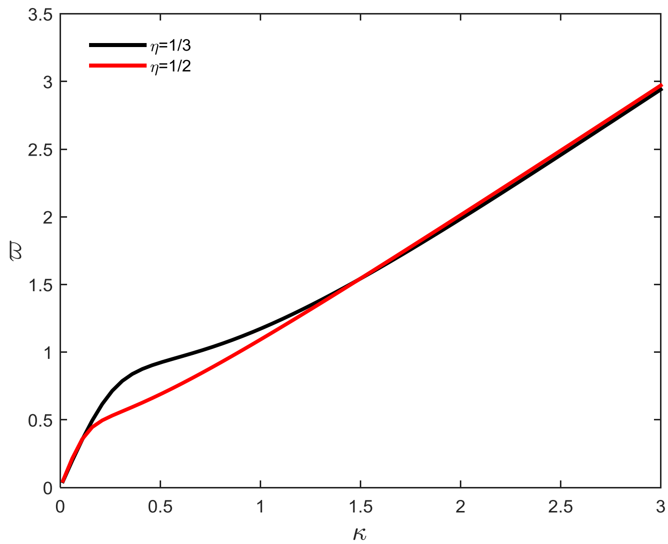

Fig. 8. Lowest frequency versus wave number for flexural waves in a hollow cylinder.

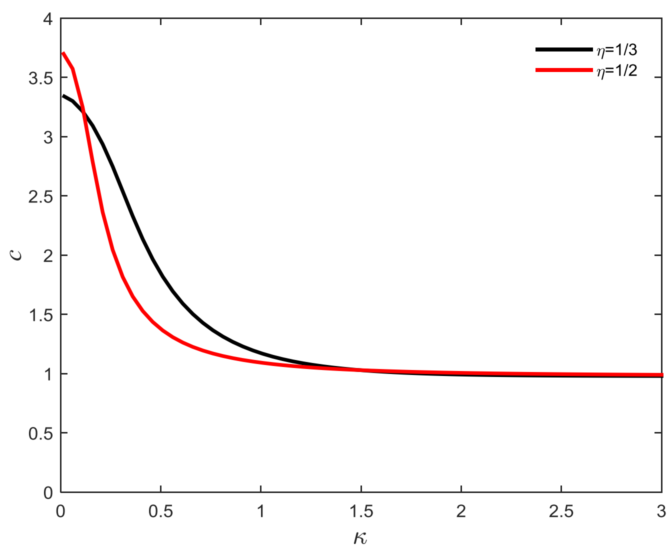

Fig. 9. Phase velocity versus wave number for flexural waves in a hollow cylinder.

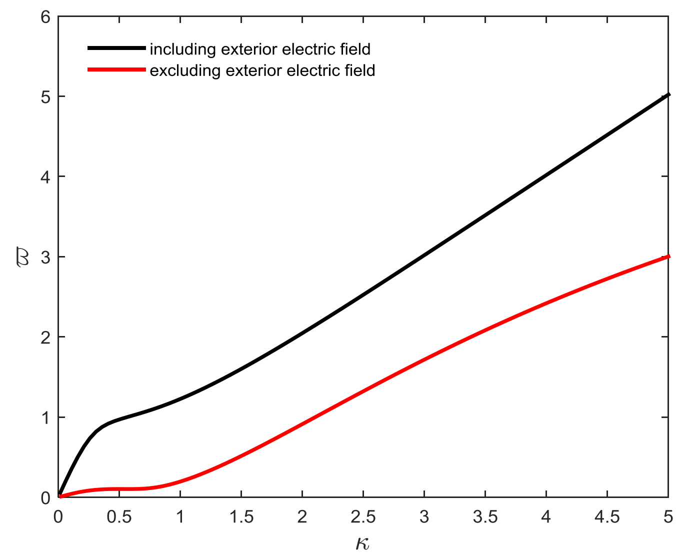

Fig. 10. Influence of external electric field on frequency for a hollow cylinder ($\eta = 1/3$).

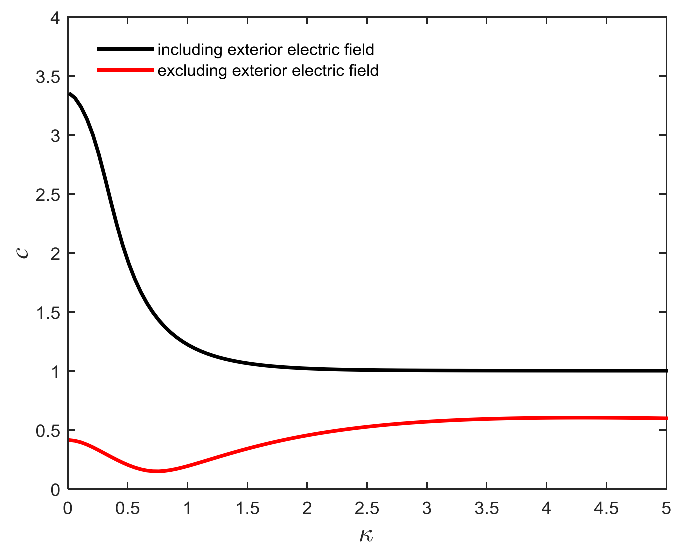

Fig. 11. Influence of external electric field on velocity for a hollow cylinder ($\eta = 1/3$).

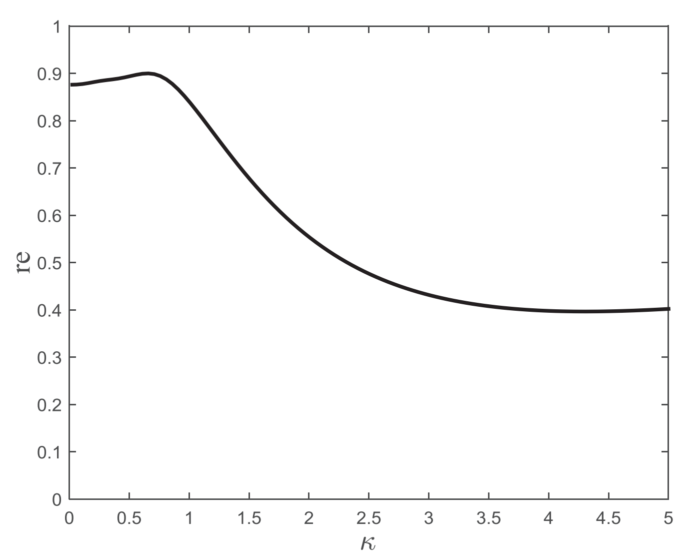

Fig. 12. The relative error between frequencies for a hollow cylinder with and without considering exterior electric field ($\eta = 1/3$).

To study its effect, Figs. 10 and 11 display respectively the frequency and velocity curves with and without considering the electric field outside the material for $\lambda_2 = 0.83$, $\gamma_1 = 1$, $\gamma_2 = 3$. Fig. 12 displays the corresponding relative error between frequencies for a hollow cylinder with and without considering the exterior electric field. The relative errors are calculated with respect to the frequency accounting for the exterior electric field, *i.e.*, $\mathrm{re} = (\varpi_{with} - \varpi_{without})/\varpi_{with}$. It is shown that the wave dispersion behavior is dramatically influenced by the electric field in the vacuum. We also point out that, due to the transfer of energy into the vacuum, the frequency and phase velocity increase when the exterior electric field is taken into consideration. Thus, the exterior electric field stiffens the electroactive cylinder to certain degree.

6. Conclusions

In this paper we applied the general nonlinear theory of electroelasticity and the associated linear incremental theory proposed recently by Dorfmann and Ogden to study the non-axisymmetric wave motion in a hollow cylinder made of soft incompressible electroactive material. In particular, uniform biasing fields were considered which may be used to control the wave propagation in hollow cylinders. Note that the control and adjustment of wave behavior becomes very important in modern wave devices, especially in the so-called phononic crystals or periodic structures.

From the analysis, we found that an isotropic electroelastic material may exhibit during the incremental motion the feature of materials with transverse isotropy when appropriate biasing fields are applied, which greatly facilitate the analysis. In fact, we managed to obtain an exact three-dimensional non-axisymmetric wave solution by introducing three displacement functions. While the earlier results in Chen and Dai (2012) are fully recovered for axisymmetric waves in a solid cylinder, the general dispersion relation can be further degenerated to that for purely elastic cylinders in the absence of any biasing field.

Numerical investigation of the fundamental modes (with the lowest frequency) of the axisymmetric and the fundamental flexural waves in the electroactive cylinder was conducted to examine the influences of the biasing fields and the geometry of the cylinder. The effect of the external electric field outside the cylinder was also studied. Results show that the applied electric field has a stiffening effect on the material, leading to increase in frequency or phase velocity of waves in the cylinder. When the wave number becomes infinitely large, the dispersion behavior becomes the same as the one for surface waves in a half-space. This is just expected. Compared with a thick-walled cylinder, a more rapid decrease of the velocity with the wave number was observed in a thin-walled cylinder, but both approaching the Rayleigh wave velocity for an electroactive half-space as $\kappa \to \infty$. Besides, the exterior electric field also indicates a stiffening effect on the electroactive cylinder, which may become significant in certain situations. These observations and discussions provide a solid theoretical basis for the design of acoustic wave devices with controllable properties.

Acknowledgments

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Appendix A. Components of the instantaneous electroelastic moduli tensors

According to Dorfmann and Ogden's theory (Dorfmann and Ogden, 2005, 2006, 2010b), the following non-zero components of the instantaneous electroelastic tensors can be obtained

$$A_{01111} = \frac{2}{\lambda_1^4}\Big\{2\Omega_{22}\big(1+\lambda_1^6\big)^2 + \lambda_1^2\big[4\Omega_{12}\lambda_1^2+\Omega_1\lambda_1^4+2\Omega_{11}\lambda_1^6+4\Omega_{12}\lambda_1^8+\Omega_2\big(1+\lambda_1^6\big)\big]\Big\},$$

$$A_{01122} = 8\Omega_{12}\big(1+\lambda_1^6\big) + \frac{4}{\lambda_1^4}\Big[(\Omega_{11}+\Omega_2)\lambda_1^8 + \Omega_{22}\big(1+\lambda_1^6\big)^2\Big],$$

$$A_{01133} = \frac{2}{\lambda_1^6}\Big\{\Omega_{12}\big(2+6\lambda_1^6\big) + 2\lambda_1^2\big[(\Omega_{11}+\Omega_2)\lambda_1^2 + 2\Omega_{22}(1+\lambda_1^6)\big], + 2D_z^2\big[2\Omega_{26}\big(1+\lambda_1^6\big) + \lambda_1^4\big(2\Omega_{16}+\Omega_{25}+\Omega_{15}\lambda_1^4+\Omega_{25}\lambda_1^6\big)\big]\Big\},$$

$$A_{03333} = \frac{2}{\lambda_1^8}\Big\{2\Omega_{11}+8\Omega_{12}\lambda_1^2+\Omega_1\lambda_1^4+8\Omega_{22}\lambda_1^4+2\Omega_2\lambda_1^6 + 2D_z^4\big(4\Omega_{66}+4\Omega_{56}\lambda_1^4+\Omega_{55}\lambda_1^8\big), + D_z^2\big[8\Omega_{16}+16\Omega_{26}\lambda_1^2+\lambda_1^4\big(4\Omega_{15}+6\Omega_6+8\Omega_{25}\lambda_1^2+\Omega_5\lambda_1^4\big)\big]\Big\}$$

$$A_{01313} = 2\lambda_1^2\big(\Omega_1 + D_z^2\Omega_6 + \Omega_2\lambda_1^2\big), \quad A_{01331} = -\frac{2}{\lambda_1^2}\Omega_2 + 2D_z^2\Omega_6\lambda_1^2,$$

$$A_{03131} = \frac{2}{\lambda_1^4}\Big\{\Omega_1 + \Omega_2\lambda_1^2 + D_z^2\big[\Omega_5\lambda_1^4 + \Omega_6\big(2+\lambda_1^6\big)\big]\Big\},$$

$$A_{01212} = \frac{2}{\lambda_1^2}\big(\Omega_2 + \Omega_1\lambda_1^4\big), \quad A_{01221} = -2\Omega_2\lambda_1^4,$$

$$\Gamma_{0113} = \frac{4D_z}{\lambda_1^6}\Big\{\Omega_{26}\big(1+\lambda_1^6\big) + \lambda_1^4\big[\Omega_{16}+\Omega_{25}\big(1+\lambda_1^6\big)+\lambda_1^4\big(\Omega_{15}+\Omega_{24}+\Omega_{14}\lambda_1^4+\Omega_{24}\lambda_1^6\big)\big]\Big\},$$

$$\Gamma_{0131} = \frac{2D_z}{\lambda_1^4}\big(\Omega_6 + \Omega_5\lambda_1^4 + \Omega_6\lambda_1^6\big),$$

$$\Gamma_{0333} = \frac{4D_z}{\lambda_1^8}\Big\{\Omega_{16} + D_z^2\big[2\Omega_{66}+3\Omega_{56}\lambda_1^4 + \lambda_1^8\big(2\Omega_{46}+\Omega_{55}+\Omega_{45}\lambda_1^4\big)\big], + \lambda_1^2\big[2\Omega_{26} + \lambda_1^2\big(\Omega_{15}+2\Omega_6+2\Omega_{25}\lambda_1^2+\Omega_{14}\lambda_1^4+\Omega_5\lambda_1^4+2\Omega_{24}\lambda_1^6\big)\big]\Big\},$$

$$K_{011} = \frac{2}{\lambda_1^2}\big(\Omega_4 + \Omega_5\lambda_1^2 + \Omega_6\lambda_1^4\big),$$

$$K_{033} = \frac{2}{\lambda_1^8}\Big\{\lambda_1^4\big(\Omega_6+\Omega_5\lambda_1^4+\Omega_4\lambda_1^8\big) + 2D_z^2\big[\Omega_{66}+2\Omega_{56}\lambda_1^4+\lambda_1^8\big(2\Omega_{46}+\Omega_{55}+2\Omega_{45}\lambda_1^4+\Omega_{44}\lambda_1^8\big)\big]\Big\},$$

where $\Omega_{ij} = \partial^2\Omega/\partial I_i \partial I_j$.

Appendix B. Some analytical results

The analytical expressions for various field variables in accordance with Eqs. (44) and (55) are given as follows:

$$\dot{T}_{0rr} = \Sigma_{rr}\cos(n\theta)\cos(kz-\omega t),$$
$$\dot{T}_{0r\theta} = \Sigma_{r\theta}\sin(n\theta)\cos(kz-\omega t),$$
$$\dot{T}_{0rz} = \Sigma_{rz}\cos(n\theta)\sin(kz-\omega t),$$
$$\dot{E}_{l0\theta} = \frac{n}{r}\bar{\phi}\sin(n\theta)\sin(kz-\omega t),$$
$$\dot{E}_{l0z} = -k\bar{\phi}\cos(n\theta)\cos(kz-\omega t),$$
$$\dot{D}_{l0r} = \Big\{e_{15}\Big[-k\Big(\frac{n}{r}\bar{\psi}-\bar{G}'\Big)+\bar{W}'\Big] - \varepsilon_{11}\bar{\phi}'\Big\}\cos(n\theta)\sin(kz-\omega t),$$
$$\dot{E}^*_{mr} = -\frac{\partial\phi^*_m}{\partial r} = -A_m Z'_n(kr)\cos(n\theta)\sin(kz-\omega t),$$
$$\dot{E}^*_{m\theta} = -\frac{1}{r}\frac{\partial\phi^*_m}{\partial\theta} = A_m\frac{n}{r}Z_n(kr)\sin(n\theta)\sin(kz-\omega t),$$
$$\dot{E}^*_{mz} = -\frac{\partial\phi^*_m}{\partial z} = -A_m k Z_n(kr)\cos(n\theta)\cos(kz-\omega t),$$
$$\dot{D}^*_{mr} = \varepsilon_0\dot{E}^*_{mr} = -A_m\varepsilon_0 Z'_n(kr)\cos(n\theta)\sin(kz-\omega t),$$

where the prime indicates derivative with respect to r, and

$$\Sigma_{rr} = (c_{11}+p)\Big(\frac{n}{r}\bar{\psi}' - \frac{n}{r^2}\bar{\psi} - \bar{G}''\Big) + c_{12}\frac{1}{r}\Big[n\Big(-\bar{\psi}'+\frac{n}{r}\bar{G}\Big)+\frac{n}{r}\bar{\psi}-\bar{G}'\Big] + c_{13}k\bar{W} + e_{31}k\bar{\phi} - \bar{p},$$

$$\Sigma_{r\theta} = c_{14}\Big(-\bar{\psi}'' + \frac{n}{r}\bar{G}' - \frac{n}{r^2}\bar{G}\Big) - (c_{15}+p)\frac{1}{r}\Big[n\Big(\frac{n}{r}\bar{\psi}-\bar{G}'\Big) - \bar{\psi}' + \frac{n}{r}\bar{G}\Big],$$

$$\Sigma_{rz} = -(c_{551}+p)k\Big(\frac{n}{r}\bar{\psi}-\bar{G}'\Big) + c_{552}\bar{W}' + e_{15}\bar{\phi}'.$$

Appendix C. Elements of d_{ij}

$$d_{14} = (c_{11}+p+\tau^*_{orr})\Big[\frac{n}{r_o}J'_n(\alpha_4 r_o) - \frac{n}{r_o^2}J_n(\alpha_4 r_o)\Big] + c_{12}\frac{1}{r_o}\Big[-nJ'_n(\alpha_4 r_o) + \frac{n}{r_o}J_n(\alpha_4 r_o)\Big],$$

$$d_{18} = (c_{11} + p + \tau^*_{orr})\left[\frac{n}{r_o}Y'_n(\alpha_4 r_o) - \frac{n}{r_o^2}Y_n(\alpha_4 r_o)\right] + c_{12}\frac{1}{r_o}\left[-nY'_n(\alpha_4 r_o) + \frac{n}{r_o}Y_n(\alpha_4 r_o)\right],$$

$$d_{1j} = -(c_{11} + p + \tau^*_{orr})Z''_n(\alpha_j r_o) + c_{12}\frac{1}{r_o}\left[\frac{n^2}{r_o}Z_n(\alpha_j r_o) - Z'_n(\alpha_j r_o)\right] + (c_{13}k\beta_{1j} + e_{31}k\beta_{2j} - \beta_{3j})Z_n(\alpha_j r_o),$$

$$d_{24} = -c_{14}J''_n(\alpha_4 r_o) - (c_{15} + p + \tau^*_{o\theta\theta})\frac{1}{r_o}\left[\frac{n^2}{r_o}J_n(\alpha_4 r_o) - J'_n(\alpha_4 r_o)\right],$$

$$d_{28} = -c_{14}Y''_n(\alpha_4 r_o) - (c_{15} + p + \tau^*_{o\theta\theta})\frac{1}{r_o}\left[\frac{n^2}{r_o}Y_n(\alpha_4 r_o) - Y'_n(\alpha_4 r_o)\right],$$

$$d_{2j} = c_{14}\left[\frac{n}{r_o}Z'_n(\alpha_j r_o) - \frac{n}{r_o^2}Z_n(\alpha_j r_o)\right] - (c_{15} + p + \tau^*_{o\theta\theta})\frac{1}{r_o}\left[-nZ'_n(\alpha_j r_o) + \frac{n}{r_o}Z_n(\alpha_j r_o)\right],$$

$$d_{34} = -(c_{551} + \tau^*_{ozz} + p)k\frac{n}{r_o}J_n(\alpha_4 r_o),$$

$$d_{38} = -(c_{551} + \tau^*_{ozz} + p)k\frac{n}{r_o}Y_n(\alpha_4 r_o),$$

$$d_{3j} = (c_{551} + \tau^*_{ozz} + p)kZ'_n(\alpha_j r_o) + c_{552}\beta_{1j}Z'_n(\alpha_j r_o) + e_{15}\beta_{2j}Z'_n(\alpha_j r_o),$$

$$d_{44} = 0, \quad d_{48} = 0, \quad d_{4j} = \frac{n}{r_o}\beta_{2j}Z_n(\alpha_j r_o) + \frac{n}{r_o}E^*_{oz}\beta_{1j}Z_n(\alpha_j r_o),$$

$$d_{54} = -k(e_{15} + D^*_{oz})\frac{n}{r_o}J_n(\alpha_4 r_o), \quad d_{58} = -k(e_{15} + D^*_{oz})\frac{n}{r_o}Y_n(\alpha_4 r_o),$$

$$d_{5j} = (e_{15} + D^*_{oz})kZ'_n(\alpha_j r_o) + e_{15}\beta_{1j}Z'_n(\alpha_j r_o) - \varepsilon_{11}\beta_{2j}Z'_n(\alpha_j r_o),$$

$$d_{19} = -\varepsilon_0 E^*_{oz}kK_n(kr_o), \quad d_{29} = 0,$$

$$d_{39} = \varepsilon_0 E^*_{oz}K'_n(kr_o), \quad d_{49} = -\frac{n}{r_o}K_n(kr_o), \quad d_{59} = \varepsilon_0 K'_n(kr_o),$$

$$d_{64} = (c_{11} + p + \tau^*_{irr})\left[\frac{n}{r_i}J'_n(\alpha_4 r_i) - \frac{n}{r_i^2}J_n(\alpha_4 r_i)\right] + c_{12}\frac{1}{r_i}\left[-nJ'_n(\alpha_4 r_i) + \frac{n}{r_i}J_n(\alpha_4 r_i)\right],$$

$$d_{68} = (c_{11} + p + \tau^*_{irr})\left[\frac{n}{r_i}Y'_n(\alpha_4 r_i) - \frac{n}{r_i^2}Y_n(\alpha_4 r_i)\right] + c_{12}\frac{1}{r_i}\left[-nY'_n(\alpha_4 r_i) + \frac{n}{r_i}Y_n(\alpha_4 r_i)\right],$$

$$d_{6j} = -(c_{11} + p + \tau^*_{irr})Z''_n(\alpha_j r_i) + c_{12}\frac{1}{r_i}\left[\frac{n^2}{r_i}Z_n(\alpha_j r_i) - Z'_n(\alpha_j r_i)\right] + (c_{13}k\beta_{1i} + e_{31}k\beta_{2i} - \beta_{3i})Z_n(\alpha_j r_i),$$

$$d_{74} = -c_{14}J''_n(\alpha_4 r_i) - (c_{15} + p + \tau^*_{i\theta\theta})\frac{1}{r_i}\left[\frac{n^2}{r_i}J_n(\alpha_4 r_i) - J'_n(\alpha_4 r_i)\right],$$

$$d_{78} = -c_{14}Y''_n(\alpha_4 r_i) - (c_{15} + p + \tau^*_{i\theta\theta})\frac{1}{r_i}\left[\frac{n^2}{r_i}Y_n(\alpha_4 r_i) - Y'_n(\alpha_4 r_i)\right],$$

$$d_{7j} = c_{14}\left[\frac{n}{r_i}Z'_n(\alpha_j r_i) - \frac{n}{r_i^2}Z_n(\alpha_j r_i)\right] - (c_{15} + p + \tau^*_{i\theta\theta})\frac{1}{r_i}\left[-nZ'_n(\alpha_j r_i) + \frac{n}{r_i}Z_n(\alpha_j r_i)\right],$$

$$d_{84} = -(c_{551} + \tau^*_{izz} + p)k\frac{n}{r_i}J_n(\alpha_4 r_i),$$

$$d_{88} = -(c_{551} + \tau^*_{izz} + p)k\frac{n}{r_i}Y_n(\alpha_4 r_i),$$

$$d_{8j} = (c_{551} + \tau^*_{izz} + p)kZ'_n(\alpha_j r_i) + c_{552}\beta_{1i}Z'_n(\alpha_j r_i) + e_{15}\beta_{2i}Z'_n(\alpha_j r_i),$$

$$d_{94} = 0, \quad d_{98} = 0, \quad d_{9j} = \frac{n}{r_i}\beta_{2i}Z_n(\alpha_j r_i) + \frac{n}{r_i}E^*_{iz}\beta_{1i}Z_n(\alpha_j r_i),$$

$$d_{104} = -k(e_{15} + D^*_{iz})\frac{n}{r_i}J_n(\alpha_4 r_i), \quad d_{108} = -k(e_{15} + D^*_{iz})\frac{n}{r_i}Y_n(\alpha_4 r_i),$$

$$d_{10i} = (e_{15} + D^*_{iz})kZ'_n(\alpha_j r_i) + e_{15}\beta_{1i}Z'_n(\alpha_j r_i) - \varepsilon_{11}\beta_{2i}Z'_n(\alpha_j r_i),$$

$$d_{610} = -\varepsilon_0 E^*_{iz}kI_n(kr_i), \quad d_{710} = 0,$$

$$d_{810} = \varepsilon_0 E^*_{iz}I'_n(kr_i), \quad d_{910} = -\frac{n}{r_i}I_n(kr_i), \quad d_{1010} = \varepsilon_0 I'_n(kr_i),$$

where, $j = 1, 2, 3, 5, 6, 7$. Especially, we have $Z(\cdot) = J(\cdot)$ for $j = 1, 2, 3$ and $Z(\cdot) = Y(\cdot)$ for $j = 5, 6, 7$. Furthermore, the notations $\alpha_{j+4} = \alpha_j$. $\beta_{n(j+4)} = \beta_{nj}(j, n = 1, 2, 3)$ have been adopted.

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